number of partitions. The major discrepancy between our computations and those of Ogusu and Tanaka appears to be around carrier densities of 10^{17} cm^{-3}, which is near maximum wave attenuation. We note that the two methods have been compared by other researchers [7], [8] in regard to accuracy and efficiency. Nevertheless, we feel that the simultaneous solution of the eigenvalues and differential equations can be more effectively performed by the multipoint boundary-value solver as described in our paper.

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Comments on "Self-Adjoint Vector Variational Formulation for Lossy Anisotropic Dielectric Waveguide"

ROLAND HOFFMANN

In the above paper,[1] the authors present a "new variational formula" and its derivation. A careful inspection of the text shows that there are a number of errors and wrong conclusions with the fatal consequence that the final variational formula [1, eq. (37)] is incorrect. The main fallacy of the authors appears to be the derivation of the adjoint solution, and the following discussion will be restricted to this point.

The authors state correctly [1, eq. (11)] that, for real inner product, the eigenvalue of the adjoint problem is $\gamma^a = -\gamma$ (while [1, eq. (12)] should read $\gamma^a = -\gamma^*$). The arguments following this equation are not complete and the conclusions are not clear. It is in fact true that there are several classes of waveguides with the property that γ as well as $-\gamma$ is a valid eigenvalue of the problem. But in contrast to the authors method, this may appear as a solution of [1, eq. (1)] as well as [1, eq. (2)] by taking into account that the electromagnetic fields, i.e., the eigenvectors are different for $+\gamma$ and $-\gamma$; hence, the matrix B differs. This property, named bidirectionality, has been thoroughly worked out in the excellent paper by McIsaac [2], where it turns out that even in the most general case of loss gyrotropic media there are classes of waveguide which exhibit this property.

In these cases, it will be possible to identify the adjoint solution with the eigenvector of the original waveguide belonging to $-\gamma$, i.e., the backward-running wave, but we are not allowed to conclude self-adjointness, as the authors obviously do by giving the condition [1, eqs. (25), (26)]

$$H^a(x,y) = H(x,y)$$

$$E^a(x,y) = E(x,y)$$

for the adjoint solution. This does not hold because the adjoint solution is the backward-running wave in the original waveguide whose fields are different from those of the wave running in the $+z$ direction with $+\gamma$.

Having drawn wrong conclusions about the adjoint fields, the authors neglect the terms with the factor γ in [1, eq. (35)]. However, these terms will not cancel, taking into account the correct adjoint solution. Thus, the final variational formula [1, eq. (37)] is wrong. No doubt it is a stationary formula, but not for solutions of the correct differential equation including the γ terms

$$\nabla_T \times \epsilon^{-1} \nabla_T \times \boldsymbol{H} + \gamma\left(\boldsymbol{u}_z \times \epsilon^{-1} \nabla_T \times \boldsymbol{H} + \nabla_T \times \epsilon^{-1} \boldsymbol{u}_z \times \boldsymbol{H}\right) + \gamma^2 \cdot \boldsymbol{u}_z \times \epsilon^{-1} \boldsymbol{u}_z \times \boldsymbol{H} - \omega^2 \epsilon_0 \mu_0 \boldsymbol{H} = 0 \qquad (1)$$

which is different from the Euler equation [1, eq. (41)] of the variational formula.

Thus, this formula will not give good approximations for the propagation constant γ by substituting trial functions for the magnetic field, nor will it give correct solutions for the magnetic field applying the Ritz procedure to the stationary formula.

Looking for reasons for the authors error, it is observed initially that they do not take into account the information given in [9] of their reference list ([4] here), where in (53) the backward-running wave has been identified as the adjoint solution, as well as in eq. (18) of their reference [10] (reference [5] here). Next, it is to be seen that they shift between three-dimensional and two-dimensional field problems in their considerations. Indeed, this can be done, but utmost care has to be taken because the properties of the corresponding operators may differ. So, while it is self-adjoint for the three-dimensional problem with a complex symmetric tensor, it is non-self-adjoint for the corresponding two-dimensional waveguide problem [3]. On the other hand, they do not try to derive the adjoint operator systematically by use of [1, eq. (19)], which will always give the correct result, commencing from the correct two-dimensional wave equation.

These properties of non-self-adjoint operators are not original. They are included in a thorough study of the electromagnetic variational principle [3]. This method has the advantage that it starts with physical reality, i.e., considering isotropic/gyrotropic, lossless/lossy media. The operators describing the physical problems are studied in detail. Their properties for three-dimensional as well as two-dimensional problems are derived for both Hermitian (complex) and symmetric (real) inner products. As one result among many, it has been found that for the problem at hand no self-adjoint formulation with symmetric (real) inner product is possible. It turns out that the only way to obtain a "variational

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[1] S. R. Cvetkovic and J. B. Davies, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 129–134, Jan. 1986.

formula for the complex propagation constant" in terms of real frequency and the three-vector $\vec{H}$ is to take into account the correct adjoint solutions, which finally yields an equation in terms of γ and γ^2.

Reply[2] *by Srboljub R. Cvetkovic and J. Brian Davies*[3]

The authors wish to thank Dr.-Ing. Hoffman for pointing out the apparent lack of clarity in [1], for drawing attention to his paper [3], and for spotting the sign error in [1, eq. (12)]. We would therefore like to take this opportunity to discuss briefly these ambiguities, as they probably led Dr.-Ing. Hoffmann to incorrectly presume some of our steps and then to draw conclusions about the overall validity of (37).

Let us look at the central criticism on which those conclusions are based, i.e., that the authors overlooked the equations relevant to their argument, namely, (53) in [4] and (18) in [5], and consequently failed to establish the correct relationship between the fields in the original and the adjoint waveguides. This is in fact not true as [1, eq. (37)] was obtained from the well-known general formulation [1, eq. (35)] by expressing in it the adjoint field in terms of the components of the original field, as indeed is given by [4, eq. (53)] and under the key assumption that the permittivity tensor is symmetric. We agree with Dr.-Ing. Hoffman that γ terms, indeed, so not *simply* cancel out; but they do, after considerable algebraic manipulation, nevertheless lead to (37).

Looking at the relationship between the original and the adjoint solutions more closely, in contrast to Dr.-Ing. Hoffmann's suggestions, no attempt was made in our paper to identify the forward-running wave in the original with the forward-running wave in the adjoint waveguide. However, the existence of self-adjointness in the two-dimensional as opposed to three-dimensional problems, and using the real inner product, was still observed (following Bresler *et al.* [5]), but only under the following conditions: that the permittivity tensor is symmetric and provided the appropriate boundary conditions in the respective waveguides are satisfied. Then the two waveguides are identical, and the authors conclude that the solutions of the original and the adjoint problems must be two identical SETS of eigenvectors, which is clearly stated in the text and expressed using (25) and (26).

On the other hand, when considering the corresponding eigenvectors individually, it was nevertheless understood that the forward-running wave in the adjoint waveguide can be identified with the backward-running wave in the original guide, as stated by Bresler *et al.* [5], and this was taken into account when obtaining (37) from (35). As mentioned, this relationship between the corresponding eigenvectors in the two guides is also given by [4, eq. (53)]. This relationship is a result of introducing z dependence into the analysis when going from three- to two-dimensional problems, and can be deduced directly from Maxwell's equations and (42) in [4]. Of course, such a relationship might still be possible in case of certain tensors that are not symmetric (see [4, eq. (51)], where the self-adjointness is not present, and obviously (37) cannot then be applied.

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Comments on "Computer-Aided Design Models for Millimeter-Wave Finlines and Suspended-Substrate Microstrip Lines"

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In the above paper,[1] Pramanick and Bhartia state in Section I that, "In this paper, closed-form equations are developed for dispersion in bilateral and unilateral finlines by using equivalent susceptances of waveguide T-junctions, and for the characteristic impedances by curve fitting to the spectral-domain results." Expressions for wave propagation in finlines described by the authors are based on:

1) the dispersion model suggested by Meier [1];
2) the solution for cutoff wavelength in an air-filled finned waveguide proposed by Burton and Hoefer [2];
3) equations for the equivalent susceptances in the bilateral (eq. (9)) and unilateral (eqs. (8) and (14)) finlines;
4) factor K (eq. (18)) for the unilateral finline, which has been found empirically by the authors.

I would like to point out that the equivalent susceptances in the bilateral and unilateral finlines, using Marcuvitz's [3] formula for the equivalent network of a waveguide T-junction, have already been described in [4] and [5] (compare (9), (8), and (14) with (4), (8), and (10) in [4]). Additionally, the authors have known the paper [4], which is given as [20] in their references.

I wish to call this to the attention of the authors of the above paper so that in future articles they may place their work in proper perspective, and properly inform their readers of the state of the art.

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